Problem 1: Using the Normal Distribution to Determine the Probability of Daily Sales Below a Certain Point

Problem: A store has normally distributed daily sales. The average daily sales = $2,000 and the daily sales standard deviation = $500, What is the probability that the sales during one day will fall below $1,000?
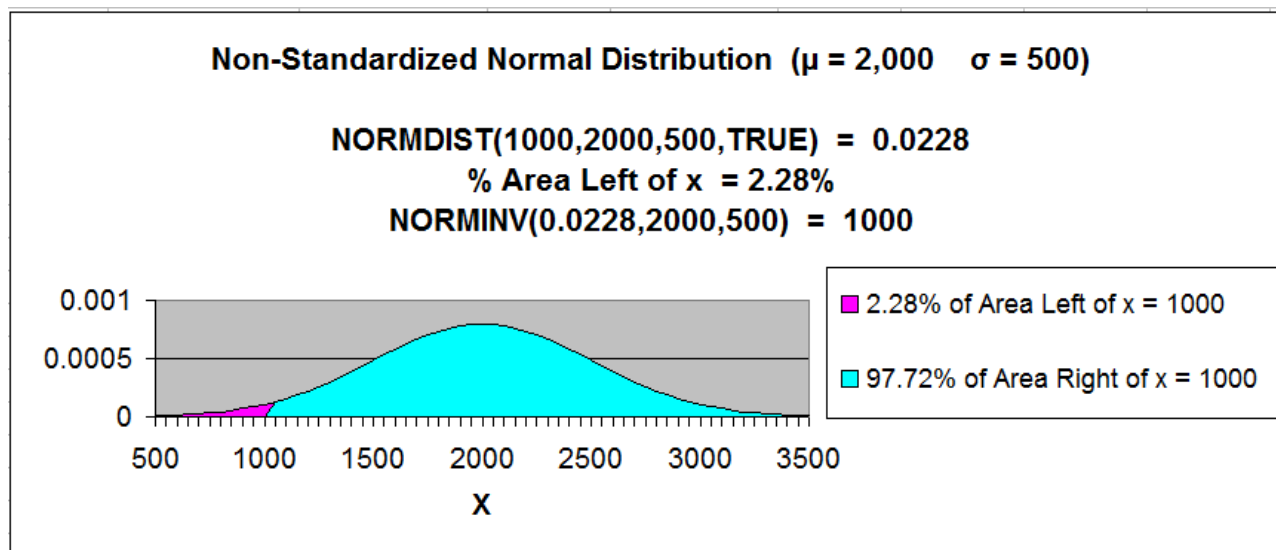
Since we are solving for a probability (area under the Normal curve) for a Normal curve that is not standardized, use NORMDIST. instead of NORMSDIST, which is used for a standardized

Normal curve.

   (Standardized Normal curve has mean = 0 and
   standard deviation = 1)

NORMDIST (1000,2000,500,TRUE) = 0.0228

Only 2.28% of the total area under this Normal curve falls to
the left of x = 1,000 if the mean = 1,000 and the standard deviation = 500

Therefore the probability is only 2.28% that daily sales will fall below 1,000.

Non-Standardized Normal Distribution (μ = 2,000   σ = 500)

NORMDIST(1000,2000,500,TRUE) = 0.0228
% Area Left of x = 2.28%
NORMINV(0.0228,2000,500) = 1000

- ■ 2.28% of Area Left of x = 1000
- ■ 97.72% of Area Right of x = 1000

Problem 2: Using the Normal Distribution to Determine the Probability that Fuel Consumption is within a Certain Range

Problem: A brand of car has a mean fuel consumption of 27 mpg with a standard deviation of 5 mpg. What percentage of the cars can be expected to have a fuel consumption of between 25 mpg AND 30 mpg? Fuel consumption is normally distributed for this population.

Population Mean = μ = "mu" = 27 mpg

Population Standard Deviation = σ = "sigma" = 5 mpg

25 mpg ≤ x ≤ 30 mpg

Probability that 25 mpg ≤ x ≤ 30 mpg = ?

mpg is Normally distributed

Normal curve is not standardized (μ ≠ 0, σ ≠ 1)

Since we are solving for a probability (area under the Normal curve)
for a Normal curve that is not standardized, use NORMDIST.

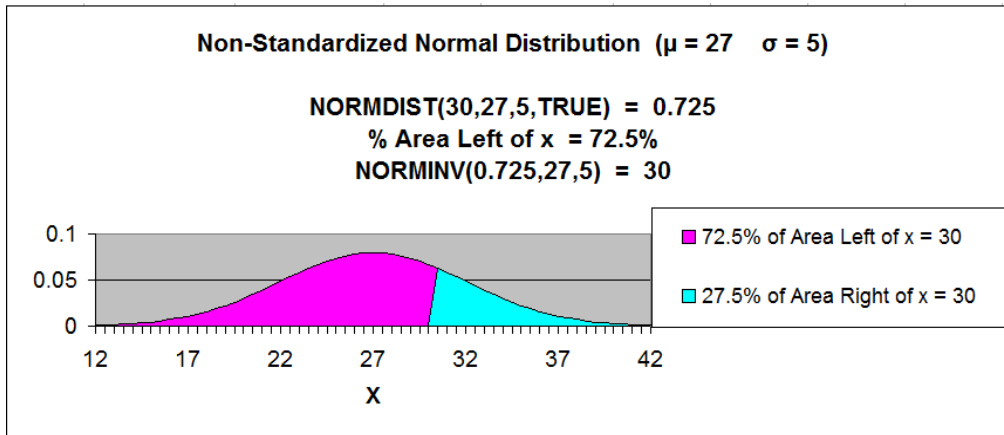Probability of a car having fuel efficiency between 25 mpg **AND** 30 mpg =

The AND statement requires that the answer contain the Union
(the elements common to both sets). This will mean subtracting
the smaller set from the bigger set to obtain only the probabilities
common to both sets.

= Probability of a car having fuel efficiency less than 30% **-** Probability of a car having fuel efficiency less than 25% =
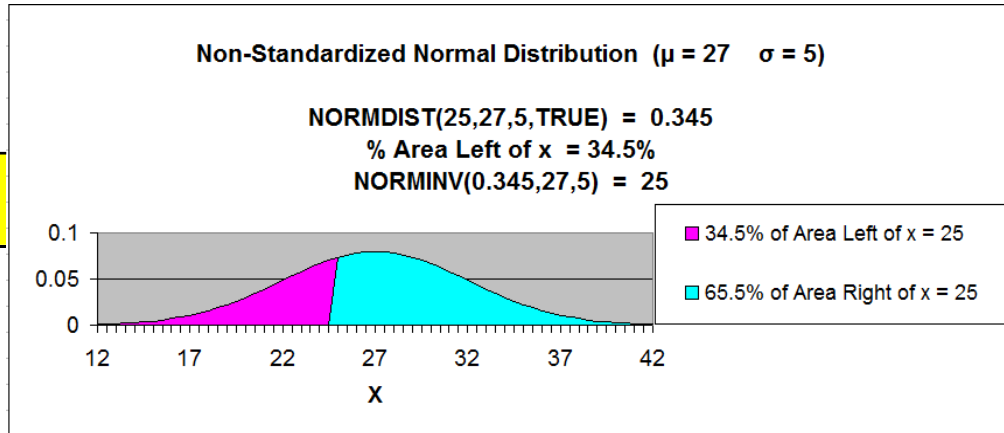
= NORMDIST(30,27,5,TRUE) **-** NORMDIST(25,27,5,TRUE) =
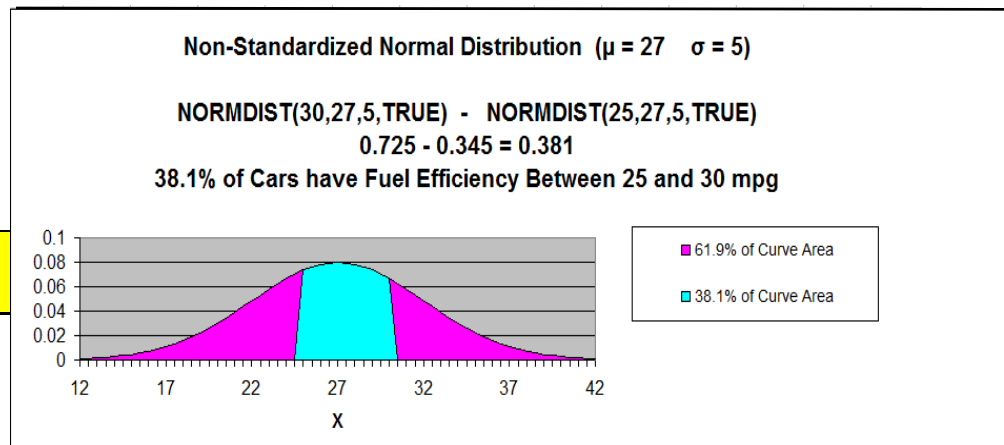
= 0.72547 **-** 0.344578 = 0.381169

38.1% of all cars can be expected to have fuel efficiency between 25 mpg and 30 mpg.

Non-Standardized Normal Distribution (μ = 27   σ = 5)

NORMDIST(30,27,5,TRUE) = 0.725
% Area Left of x = 72.5%
NORMINV(0.725,27,5) = 30

- 72.5% of Area Left of x = 30
- 27.5% of Area Right of x = 30

−

Non-Standardized Normal Distribution (μ = 27   σ = 5)

NORMDIST(25,27,5,TRUE) = 0.345
% Area Left of x = 34.5%
NORMINV(0.345,27,5) = 25

- 34.5% of Area Left of x = 25
- 65.5% of Area Right of x = 25

=

Non-Standardized Normal Distribution (μ = 27   σ = 5)

NORMDIST(30,27,5,TRUE) - NORMDIST(25,27,5,TRUE)
0.725 - 0.345 = 0.381
38.1% of Cars have Fuel Efficiency Between 25 and 30 mpg

- 61.9% of Curve Area
- 38.1% of Curve Area

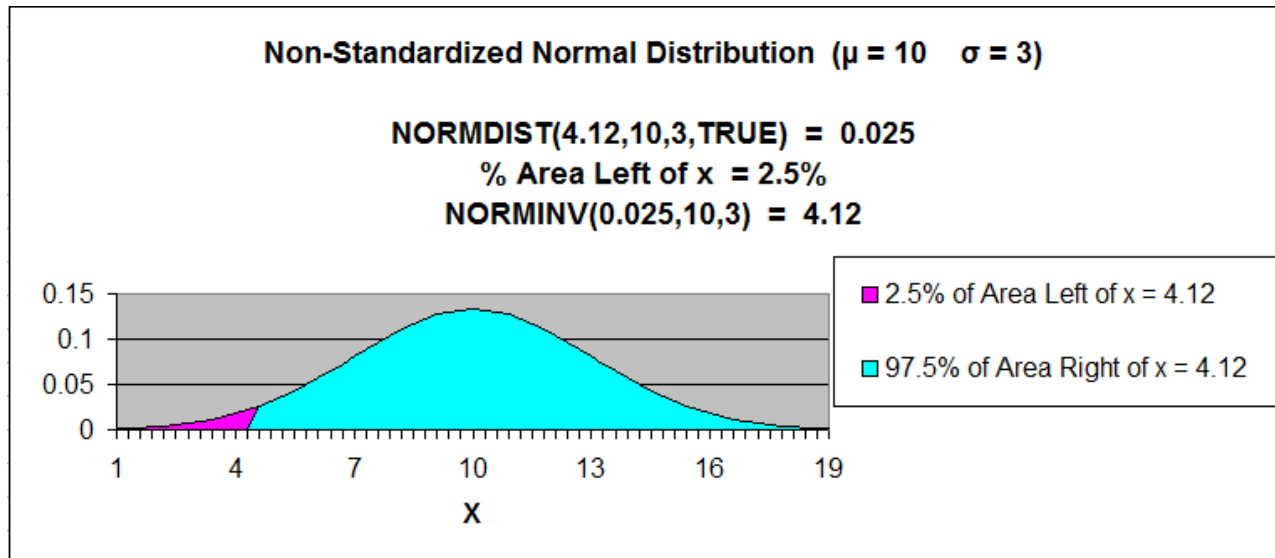**Problem 3: Using the Normal Distribution to Determine the Upper Limit of Delivery Time**

Problem: A company's package delivery time is normally distributed with a mean of 10 hours and a standard deviation of 3 hours. What delivery time will be beaten by only 2.5% of all deliveries?

If solving for x given a probability of x for a Normal curve that is not standardized, use NORMINV.

Note that NORMINV calculates the area to the left of x, so the problem parameters must be written calculating a probability less than x.

NORMINV(**0.025**,10,3) = 4.12

Only 2.5% of all package delivery times will be quicker than 4.12 hours.

Non-Standardized Normal Distribution (μ = 10   σ = 3)

NORMDIST(4.12,10,3,TRUE) = 0.025
% Area Left of x = 2.5%
NORMINV(0.025,10,3) = 4.12

■ 2.5% of Area Left of x = 4.12

■ 97.5% of Area Right of x = 4.12

X

In this chart, x = Delivery time measured in hours.

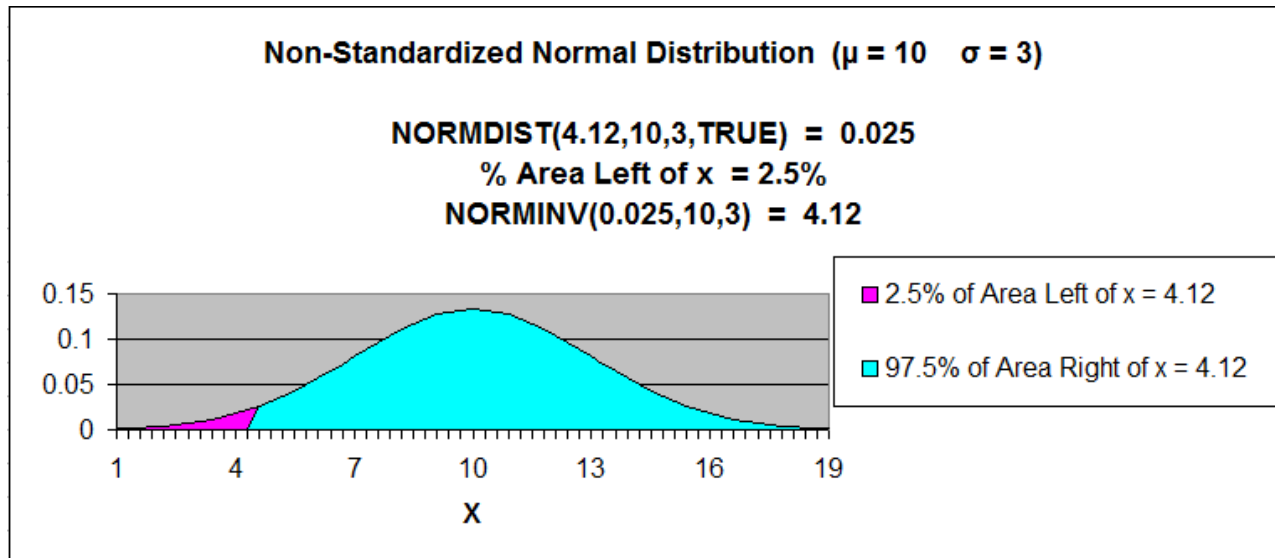**Problem 3: Using the Normal Distribution to Determine the Upper Limit of Delivery Time**

> **Problem: A company's package delivery time is normally distributed with a mean of 10 hours and a standard deviation of 3 hours. What delivery time will be beaten by only 2.5% of all deliveries?**

If solving for x given a probability of x for a Normal curve that is not standardized, use NORMINV.

Note that NORMINV calculates the area to the left of x, so the problem parameters must be written calculating a probability less than x.

NORMINV(**0.025**,10,3) = 4.12

Only 2.5% of all package delivery times will be quicker than 4.12 hours.

Non-Standardized Normal Distribution (μ = 10   σ = 3)

NORMDIST(4.12,10,3,TRUE) = 0.025
% Area Left of x = 2.5%
NORMINV(0.025,10,3) = 4.12

- 2.5% of Area Left of x = 4.12
- 97.5% of Area Right of x = 4.12

In this chart, x = Delivery time measured in hours.

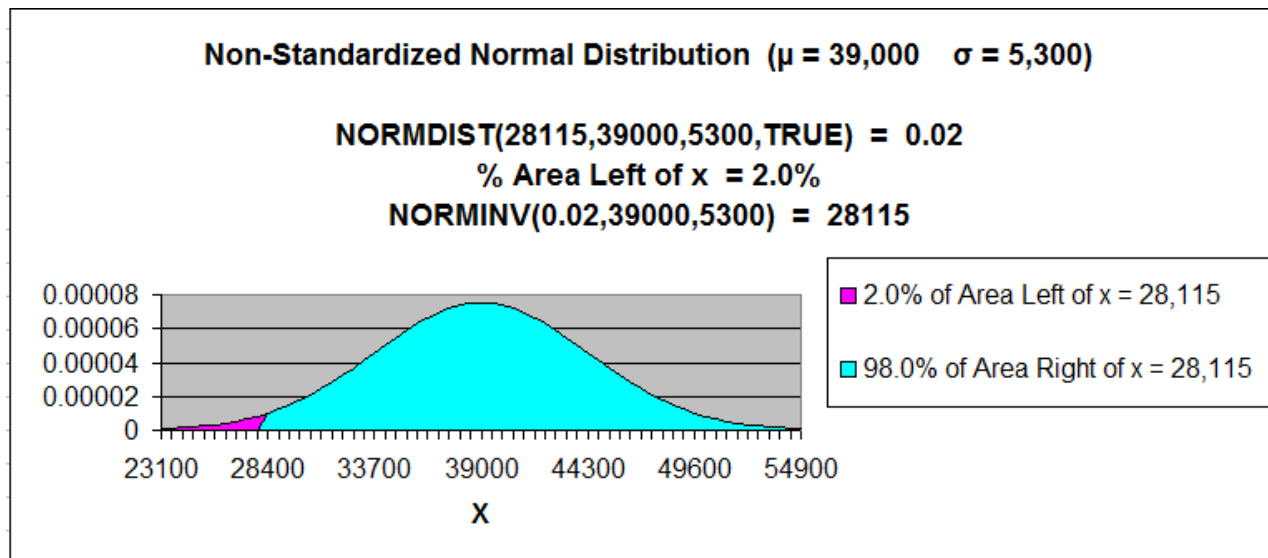**Problem 4: Using the Normal Distribution to Determine
the Lower Limit of Tire Life**

**Problem: A tire company makes a tire with a normally
distributed tread life that has a mean of 39,000 miles
and standard deviation of 5,300 miles. What tread life
would be exceeded by 98% of all tires?**

(If solving for x given the probability of x for a Normal curve
that is not standardized, use NORMINV)

Note that **NORMINV calculates the area to the left of x**,
so the problem parameters must be written calculating a
probability less than x. The problem is originally
worded asking for the probability of greater than x.

NORMINV(**0.02**,39000,5300) = 28,115

Only 2% of all tires will wear out before 28,115 miles..

Non-Standardized Normal Distribution (μ = 39,000   σ = 5,300)

NORMDIST(28115,39000,5300,TRUE) = 0.02
% Area Left of x = 2.0%
NORMINV(0.02,39000,5300) = 28115

- 2.0% of Area Left of x = 28,115
- 98.0% of Area Right of x = 28,115

Problem 5: Using the Normal Distribution to Determine the Boundaries of a Range of Tire Life

Problem: A tire company makes a tire with a normally distributed tread life that has a mean of 39,000 miles and standard deviation of 5,300 miles. What would the range of tread life be that 95% of all tires would wear out in?

Population Mean = μ = "mu" = 39,000 miles

Population Standard Deviation = σ = "sigma" = = 5,300 miles

x_1 = ? x_2 = ?

Left Boundary

Probability that (tread life $\leq x_1$) = 2% = **0.025**

And

Right Boundary

Probability that (tread life $\leq x_2$) = 97.5% = **0.975**

Tread life is Normally distributed

Normal curve is not standardized ($\mu \neq 0$, $\sigma \neq 1$)

If solving for x given a probability of x for a Normal curve
that is not standardized, use NORMINV

Note that **NORMINV calculates the area to the left of x**,
so the problem parameters must be written calculating
a **probability less than x**.

A Range is normally bounded on its upper and lower
ends. We arbitrarily chose to have the 95% range
bounded with 2.5% of tires outside on the upper and
2.5% of tires outside on the lower end. We could
have chosen any combination that would have 5%
of tires outside, such as 1% on top and 4% on the
bottom.

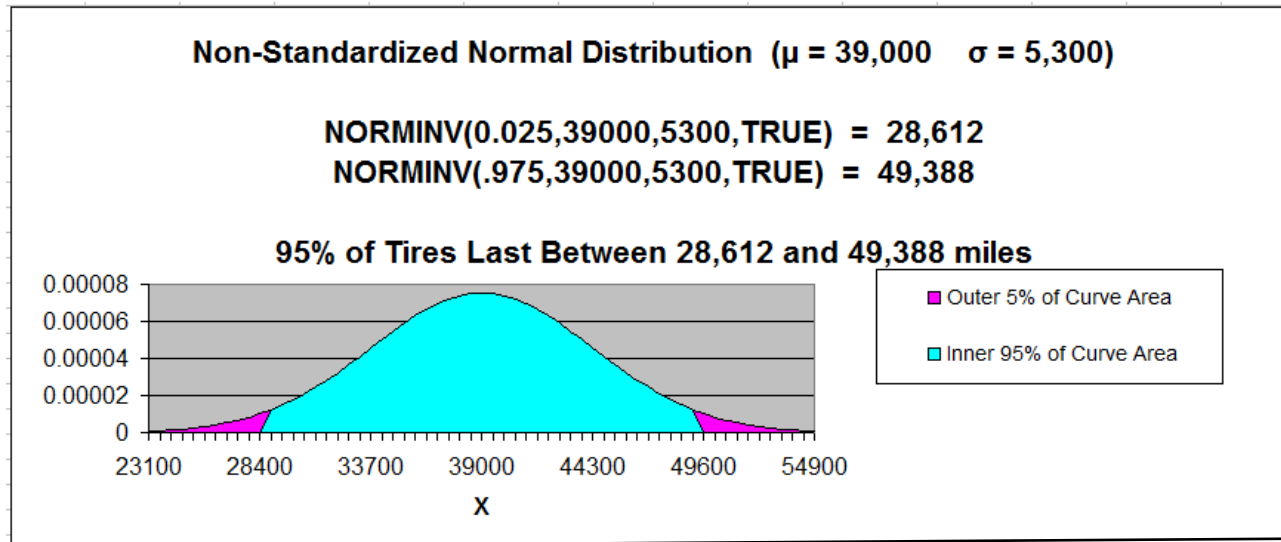## Calculation of the left boundary:

NORMINV(**0.025**,39000,5300) = 28,612

   Only 2.5% of all tires will wear out before **28,115** miles.

Calculation of the right boundary:

NORMINV(**0.975**,39000,5300) = **49,388**

   Only 2.5% of all tires will wear out after 49,388 miles..
   (97.5% of all tires will wear out before 48,388 miles)

So, 95% of tires will wear out in the range of **28,612** miles to **49,388** miles.

Non-Standardized Normal Distribution (μ = 39,000   σ = 5,300)

NORMINV(0.025,39000,5300,TRUE) = 28,612
NORMINV(.975,39000,5300,TRUE) = 49,388

95% of Tires Last Between 28,612 and 49,388 miles

Problem 6: Using the Normal Distribution to Determine the Probability of a Pumpkin's Weight Being in 1

of 2 Ranges

Problem: Assume that X, the weight of pumpkins on a farm, is normally distributed with a mean of 20 pounds and a standard deviation of 1 pound. A pumpkin is selected at random, and its weight is noted. What is the probability that the weight will be either more than 21.50 pounds OR less than 18.5 pounds?

Population Mean = μ = "mu" = 20 lbs

Population Standard Deviation = σ = "sigma" = = 1 lb

18.5 lbs ≥ x   OR   x ≥ 21.5 lbs mpg

Probability that 18.5 lbs ≥ x   OR   x ≥ 21.5 lbs mpg = ?

weight is Normally distributed

Normal curve is not standardized (μ ≠ 0, σ ≠ 1)

Since we are solving for a probability (area under the Normal curve)
for a Normal curve that is not standardized, use NORMDIST.

Probability of a pumpkin weighing less than 18.5 lbs **OR** more than 21.5 lbs =

The OR statement requires that the answer contain all elements
of both sets). This will mean adding both sets together and then
subtracting any repeating elements. Both sets are mutually
exclusive so no elements are repeating in both sets. We
therefore just add both sets together.

The probability of a pumpkin weighing **<u>less than</u>** 18.5 lbs = NORMDIST(18.5,20,1,TRUE) = **0.066807** = 6.67%

OR

The probability of a pumpkin weighing **<u>more than</u>** 21.5 lbs = **1 -** NORMDIST(21.5,20,1,TRUE)

$$= 1 - 0.933193 = \mathbf{0.066807}$$

$$= 6.67\%$$

= Probability of a pumpkin weighing **<u>less than</u>** 18.5 lbs **+** Probability of a pumpkin weighing **<u>more than</u>** 21.5 lbs. =

= NORMDIST(18.5,20,1,TRUE) **+** [**1 -** NORMDIST(21.5,20,1,TRUE)] =

= **0.066807** **+** **0.066807** = 0.133614 = 13.36%

So the probability that the pumpkin weighs less than 18.5 lbs or more than 21.5 lbs = 13.36%